Memo

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From: Christopher Settanni
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Subject: Analytical Analysis

Introduction

This memo outlines the process of calculating the moment exerted on a radome due to pressure during supersonic flight. The team needs a detailed estimate of the force exerted on a radome to calculate the largest moment a radome test fixture needs to accommodate. This analysis uses several assumptions about the size, shape, speed and geometry of the radome to compute the moment on a radome due to aerodynamic pressure.

This analysis assumes that the radome is travelling within the range of Mach 2 to Mach 5. The analysis also assumes the missile is flying at an elevation near sea level. This assumption affects the missile speed (local Mach numbers vary according to altitude) [1].

Background

Aircraft operating above Mach 0.3 are subject to compressible fluid dynamics [1]. The approximation of constant density is no longer valid. Therefore, density depends on the aircraft geometry and the Mach number. Changes in the direction of flow at supersonic speeds result in shockwaves and expansion fans. These discontinuities yield changes in pressure, temperature, density, and Mach number [1]. Shockwaves cause the Mach number to lower from a supersonic value to a value below unity (Figure 1) [1].

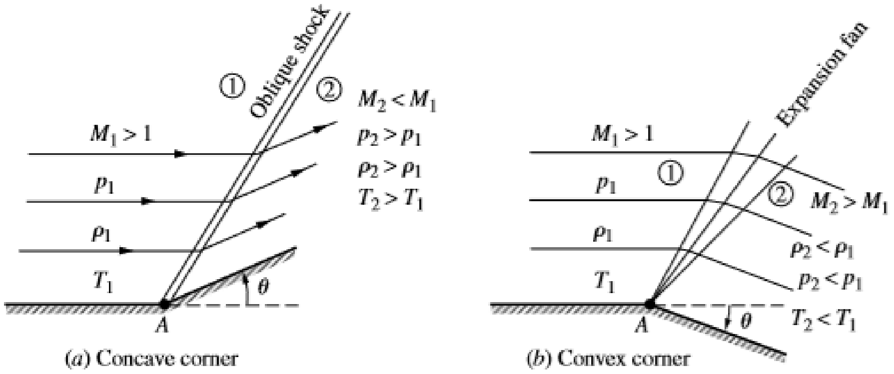

Figure 1: Oblique shockwaves and expansion fans [1].

Expansion fans are created by an increase in cross sectional area of a flow, leading to an increase in the Mach number and a decrease in pressure [1].

Assumptions

The test fixture needs to replicate the flight conditions of an actual missile during supersonic flight. Liberal estimates for radome size, Mach number, and angle of attack were used to estimate the greatest moment the test fixture needs to generate. The Radome geometry and angular momentum were also simplified for this program. Each assumption is listed below. Conservative assumptions are denoted by a black dot, while a hollow circle precedes non-conservative assumptions.

- This simulation assumes that the radome will operate at Mach 5
- The radome angle of attack will be 20 degrees
- The ambient pressure will be 1.013 bar
- The pressure distribution is two dimensional
    - Pressure on the radome is assumed to be equal to the ambient pressure in each region
    - The radome is conically shaped
    - The radome has no angular momentum

Each conservative assumption leads to a larger maximum moment on radome, while non-conservative values lower the value for the maximum moment experienced by the radome. Additionally, the two-dimensional approximation is known to be a significant source of error. The diagram below shows the transition from supersonic flow (M_1) to subsonic flow (M_2) across a shock wave (double line) [1]. Although a cone and a wedge both have a half angle (θ) of 20°, the shockwave angle (β) experiences a three-dimensional relief effect over a cone vs a wedge [1].

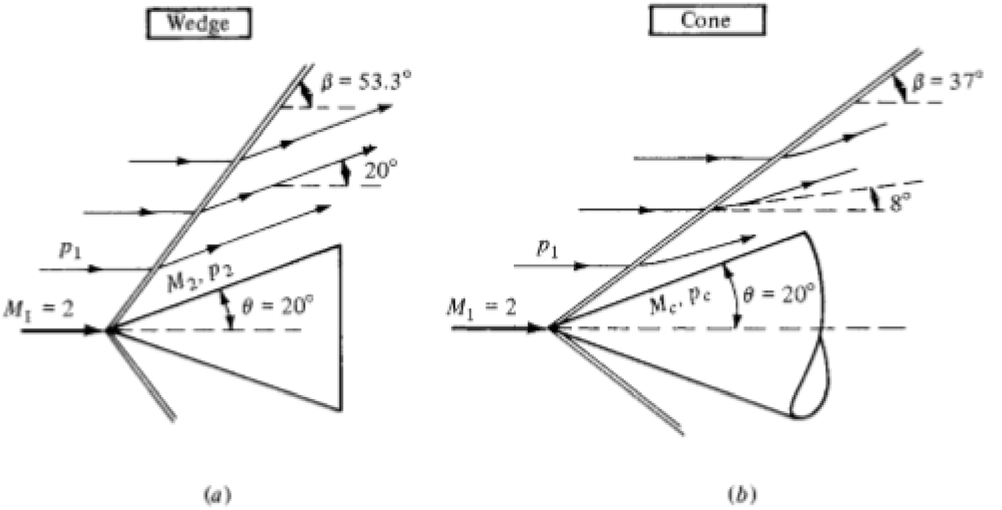

Figure 2: 2D versus 3D flow [1]

Because the streamlines are deflected less in the three-dimensional geometry, the wedge approximation will provide a larger pressure estimate for the radome than the conical approximation.

Methods and Calculations

For this analysis, a MATLAB program is used to evaluate the moment on a radome. Calculating the Pressure difference on the radome consists of four main steps. First, preliminary data is collected. This data includes the length of the radome and the half angle (20°), of the radome. The freestream flow crosses an expansion fan to reach the top (leeward) region of the radome. The freestream flow also crosses a shockwave to reach the bottom (windward) region of the radome (Figure 3) [1].

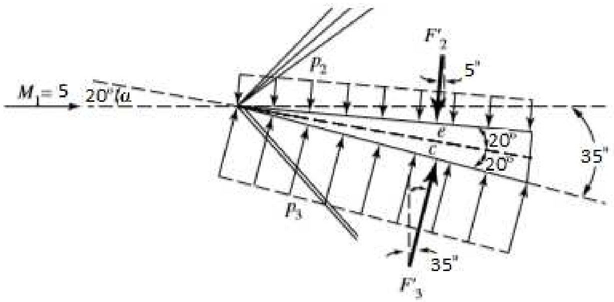

Figure 3: Pressure Diagram (Dimensions modified from original values)

After data has been collected, the program solves for the pressure at the windward and leeward sides of the radome. Next, a two-dimensional moment is calculated using the pressure difference. The freestream Mach number (M_1), angle of attack (α) are also required to compute properties at the leeward and windward sides of the radome [1]. The expansion angle (ν) is calculated using the specific heat ratio (γ) and the Mach number (M) using Equation 1 [1].

$$\nu(M) = \frac{\gamma + 1}{\gamma - 1} \tan^{-1} \sqrt{\frac{\gamma + 1}{\gamma - 1}(M^2 - 1)} - tan^{-1} \sqrt{M^2 - 1} \qquad \text{(Equation 1)}$$

The expansion angle is calculated explicitly for the freestream Mach number, then the leeward angle (θ) is used to solve for the Mach number after the expansion fan (M_2) using Equation 1 and Equation 2 [1].

$$\theta = \nu(\nu(M_1) \qquad \text{(Equation 2)}$$

Next pressure above the fan is calculated using the Mach number after the fan (M_2), freestream Mach number (M_1), and the freestream pressure (p_1) using the isentropic pressure relation across an expansion fan (Equation 3) [1].

$$\frac{p_2}{p_1} = \frac{1 + \left[\frac{\gamma - 1}{2}\right] M_1^2}{1 + \left[\frac{\gamma - 1}{2}\right] M_2^2} \qquad \text{(Equation 3)}$$

The next major step involves calculating the pressure on the windward side of the wedge. The Shockwave angle (β) is calculated implicitly, using the θ-β-M relation [1]. Equation 4 relates the Mach number, shockwave angle (β), and the angle of incidence (θ).

$$tan\theta = \cot\beta \frac{M_1^2 \sin^2\beta - 1}{M_1^2(\gamma + \cos^2 2\beta) + 2} \qquad \text{(Equation 4)}$$

The subsonic Mach number (M_1) is calculated from its normal component ($M_{n,2}$) and the shockwave angle (β) with trigonometric relationships [1]. The pressure on the windward side of the wedge (p_3) is derived from the freestream pressure (p_1), Mach number, and specific heat ratio (γ) using the pressure relationship across shockwaves (Equation 5) [1].

$$\frac{p_3}{p_1} = 1 + \frac{2\,\gamma + 1}{\gamma}(M_{n,1}^2 - 1) \tag{Equation 5}$$

Lastly, the two-dimensional pressure is integrated through a circular base profile. The following pseudocode describes the steps the program takes to calculate the moment created by pressure on the radome.

```
%Input Data
%Solve for the expansion angle (nu)
    %Find the expansion across leeward angle
    %Solve for the expansion Mach Number
    %Solve for the pressure after the expansion fan
%Calculate the shockwave angle on the windward side of the radome
    %Solve for the shock angle of the windward face
    %Solve for the pressure ratio across the windward shockwave
    %Calculate the pressure across the windward side of the wedge
    %Estimate the moment on the section of the radome
%Integrate the two-dimensional moment across the radome base
```

This code utilizes equations to solve for the freestream pressures, Mach numbers, and angles for shockwaves and expansion fans rather than hard coded values from tables. This allows the code to adapt to a range of inputs rather than describe a single scenario (Appendix A).

Results

The flight conditions for the radome yielded a maximum theoretical moment of 374 kNm (Appendix A). It is important to note that the radome size has a drastic effect on the moment exerted on the radome. Reducing the length to 1 m brought the moment on the radome down to 10kNm. This relationship is important to keep in mid for future design specifications. Additionally, this program is functional for Mach numbers between 1.5 and 5. The program will work for higher values, but the assumptions no longer hold [1].

Conclusion

The test fixture needs to be designed to accommodate this load on a consistent basis. Replicating a maximum loading of this nature carries implications for the material selection, power supplied to the test fixture, and creep resistance of the fixture. Alternatively, if the test fixture cannot meet the required specification, either the assumptions for this analysis must refined, or the team must reduce the range of speeds and radome sizes the test fixture can accommodate.

References

[1] J. D. Anderson, Fundamentals of Aerodynamics, New York: McGraw-Hill, 2011.

Appendix A:

Matlab Code and Results

```
clear all
clc
%Input Data

%Length in meters
length = 2.5;

%Freestream Mach number
M1 = 5;

%Freestream pressure (Pa)
p1 = 101325;

%Angle of attack
att_ang = 20;

%Half angle
half_ang = 15;

%Diameter in meters
diameter = 2*length*tand(half_ang);

%Top angle
top_ang = att_ang - half_ang;

%Bottom angle
bott_ang = half_ang + att_ang;

%Expansion coefficients
gamma = 1.4;
c_exp1 = sqrt((gamma+1)/(gamma-1));
c_exp2 = sqrt((gamma-1)/(gamma+1));

%Solve for the expansion angle (nu)
nu = @(M) c_exp1*( atand(c_exp2 * sqrt(M^2 - 1))) - atand(M^2 - 1);

%Expansion aross top angle
nu2 = nu(M1) + top_ang;

%Zero value function of expansion angle
exp_fun = @(M) c_exp1*( atand(c_exp2 * sqrt(M^2 - 1))) - atand(M^2 -
1) - nu2;

%Solve for the expansion Mach Number
M2 = fzero(exp_fun , M1);
%Isentropic Expansion (p_0,1 = p_0,2)
```

```
%Expansion pressure coefficient
p_coeff_exp = (gamma - 1)/2;

%Find pressure ratio
p_rat_1_2 = (( 1 + p_coeff_exp * M1^2)/( 1 + p_coeff_exp *
M2^2))^(gamma/(gamma-1));

%Solve for the pressure after the expansion fan
p2 = p1*p_rat_1_2;

%Calculate the Shockwave angle on the bottom of the radome
shock_ang_fun = @(M,Beta,theta) 2/tand(Beta)*(M^2*sind(Beta)^2-1)...
     /(M^2*(gamma+cosd(2*Beta)+2) - sind(theta));

%Solve the Mach, theta, Beta function in terms of Beta
beta_sol_fun = @(Beta) shock_ang_fun(M1,Beta,bott_ang);

%Solve for the shock angle of the bottom face
Beta3 = fzero(beta_sol_fun , att_ang);

%Solve for the freestream Mach number normal to shockwave on the
bottom
Mn1 = M1 * sind(Beta3);

%Solve for the pressure ratio across the bottom shockwave
p_rat_1_3 = 1 + (2 * gamma)/(gamma + 1)  * (Mn1^2 - 1);

%Calculate the pressure across the bottom of the wedge
p3 = p1 * p_rat_1_3;

%Estimate the moment on the section of the radome
net_load = p3*cosd(15) - p2*cosd(15);
Differential_moment = @(length) net_load *length;
Two_D_Moment = integral(Differential_moment,0,length);

%Integrate the 2-Dimensional moment across the radome base
Moment_per_width = @(width) 2 * Two_D_Moment * sqrt( diameter^2 -
width.^2 );
total_moment = integral(Moment_per_width,0,diameter);

%Convert the Moment to kNm
Moment = total_moment/1000;

%Print Results
fprintf('For a radome %2.1f m long with a %2.2f m diameter,\n
travelling at Mach%2.0f ',length,diameter,M1)
fprintf('at an agle of attack of %d degrees,\n the moment on the
radome will be %4.0f KNm.\n',att_ang, Moment)
```

```
Command Window:
For a radome 2.5 m long with a 1.34 m diameter,
 travelling at Mach 5 at an angle of attack of 20 degrees,
 the moment on the radome will be 374 kNm.

For a radome 1.0 m long with a 0.54 m diameter,
 travelling at Mach 5 at an angle of attack of 20 degrees,
 the moment on the radome will be   10 kNm.
```